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# Statistical Analysis of CFD Solutions

--- Case 1 ---

$M = 0.75$ ,  $CL = 0.5$

WB, CMF & WBPN, CMF

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2<sup>nd</sup> AIAA APA Drag Prediction Workshop  
Walt Disney Hilton, Orlando, FL  
June 21-22, 2003



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- Definition of a “collective”
- Method for analyzing the collective
- Comparison of results from DPW I and DPW II
- Grid convergence for the collective
- Conclusions and Recommendations



- A collective is ... a long sequence of observations for which there are sufficient reasons to believe that the relative frequency of the observed attribute would tend to a fixed limit if the observations were indefinitely continued.
- It is ... obvious that in trying to take into account *all* the properties of an individual, we shall finally arrive at the stage of finding no other members of the collective at all, and the collective will cease to exist altogether.



- There are three requirements for the existence of a collective and, hence, meaningful statistics:
  - A group of individuals who are more alike than not (e.g. all RANS)
  - The individuals differ in ways that are not accounted for (e.g. grid type, coding approaches, turbulence models, etc.)
  - The key properties of the collective do not change with time.



- Compare DPW I with DPW II (WB-MG)
  - Reduction in spread?
  - Reduction in scatter of “core” solutions?
- Grid Convergence for nested solutions
  - Reduction in spread?
  - Reduction in scatter of “core” solutions?
  - Significant changes in medians?



# Solution Statistics

# DPW

	DPW I	DPW II WB-MG	DPW II Nested
Solutions	38	20	15
Authors	18	19	14
Institutions	14	15	13
Codes	13	17	14



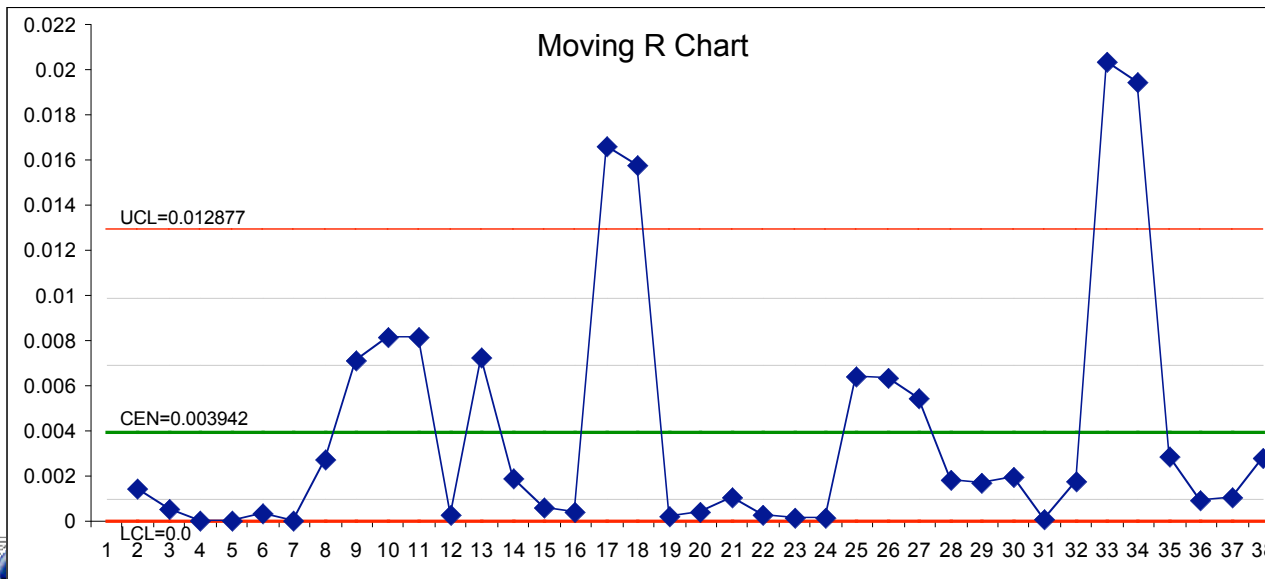
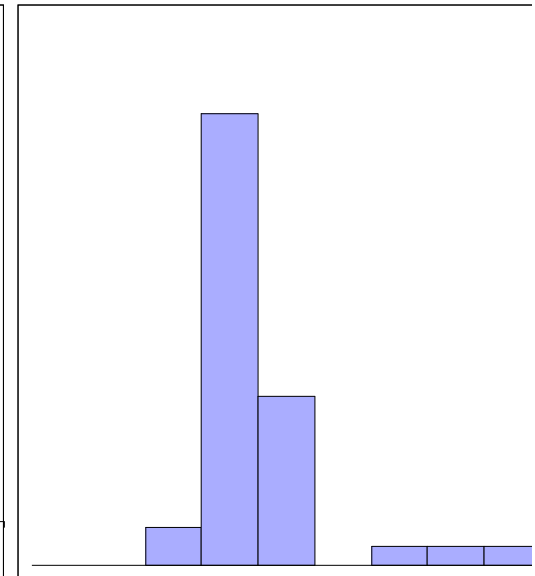
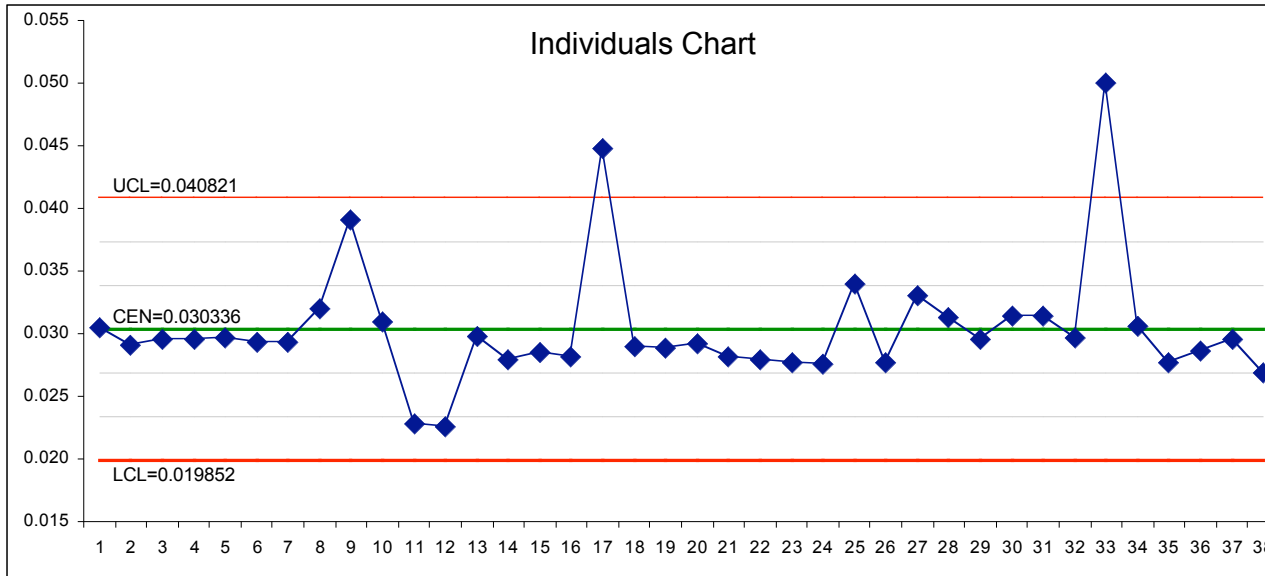
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# Comparison of DPW I to DPW II For WB-Medium Grid



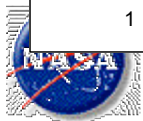
# DPW I – Case 1 (CD\_TOT)

DPW



## Before

The average moving range gives a robust estimate of the population standard deviation.

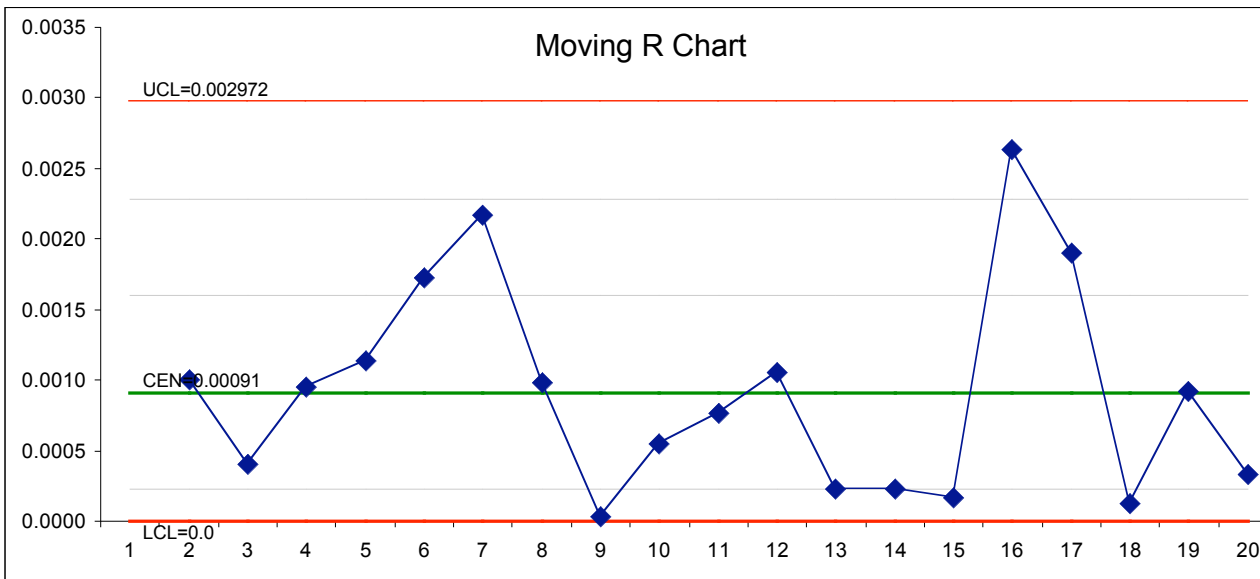
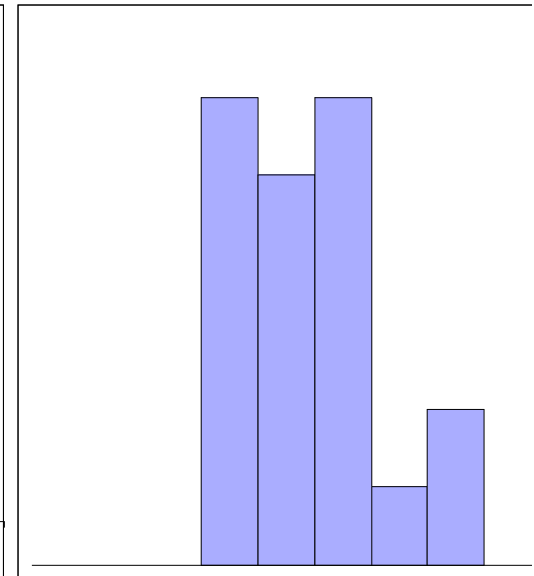
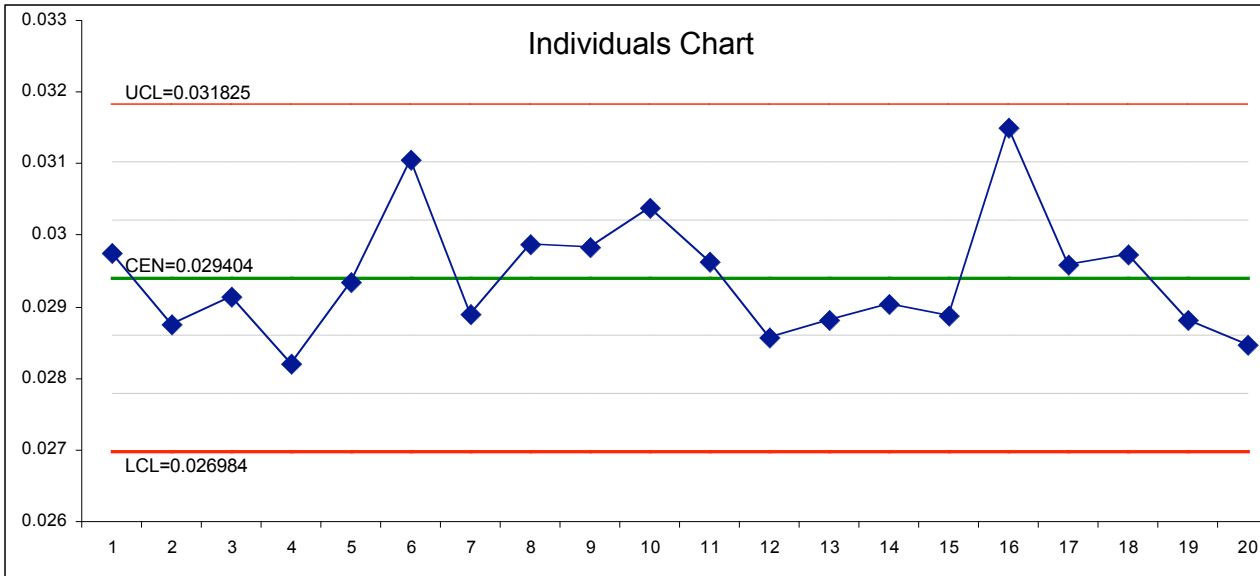


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# DPW II – Case 1 (CD\_TOT)

DPW



**After (WB-MG)**

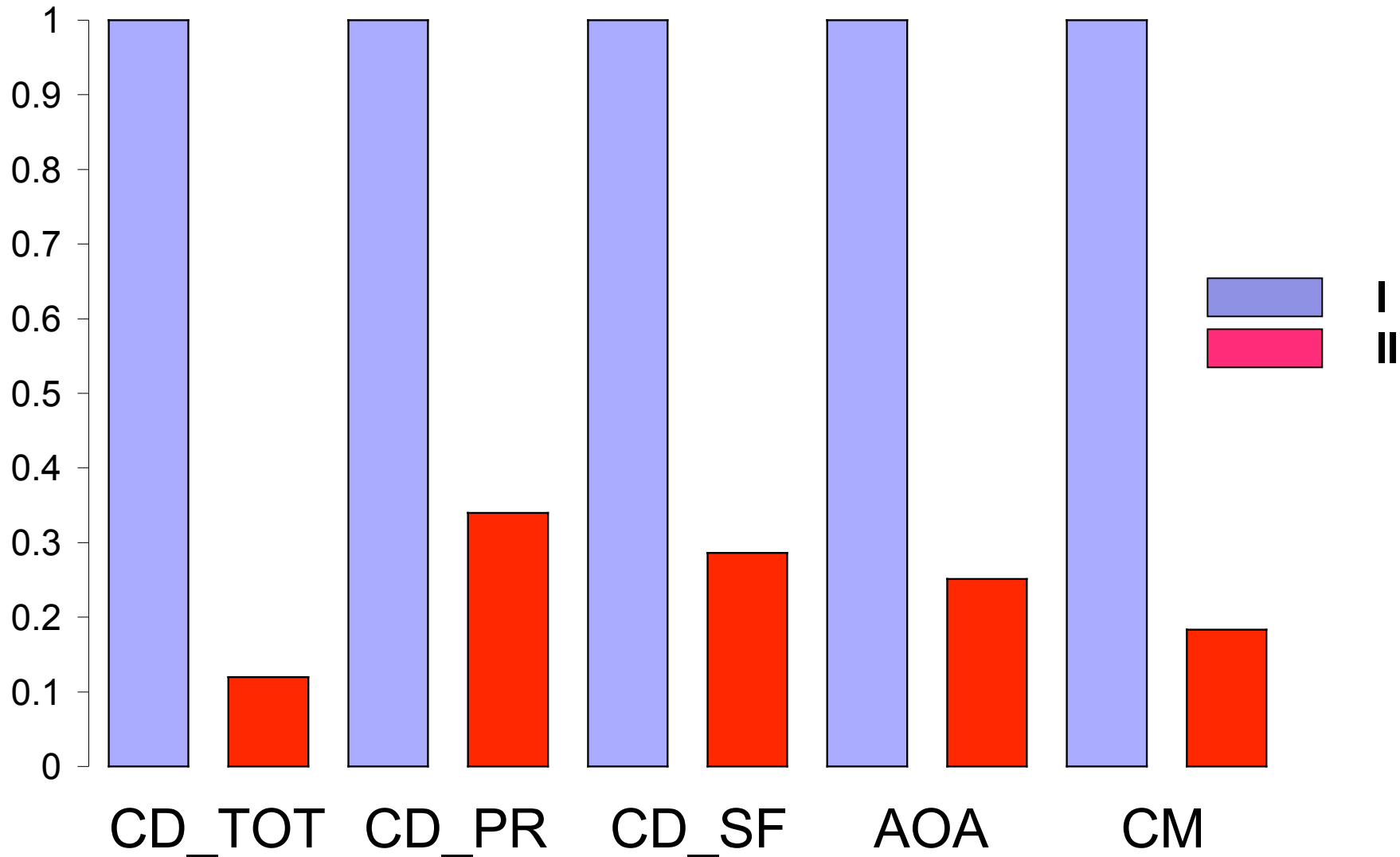
	DPW I	DPW II
R	274 cts	33 cts
AMR	39 cts	9 cts



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# Spread (all data) for WB (med. grid)

DPW



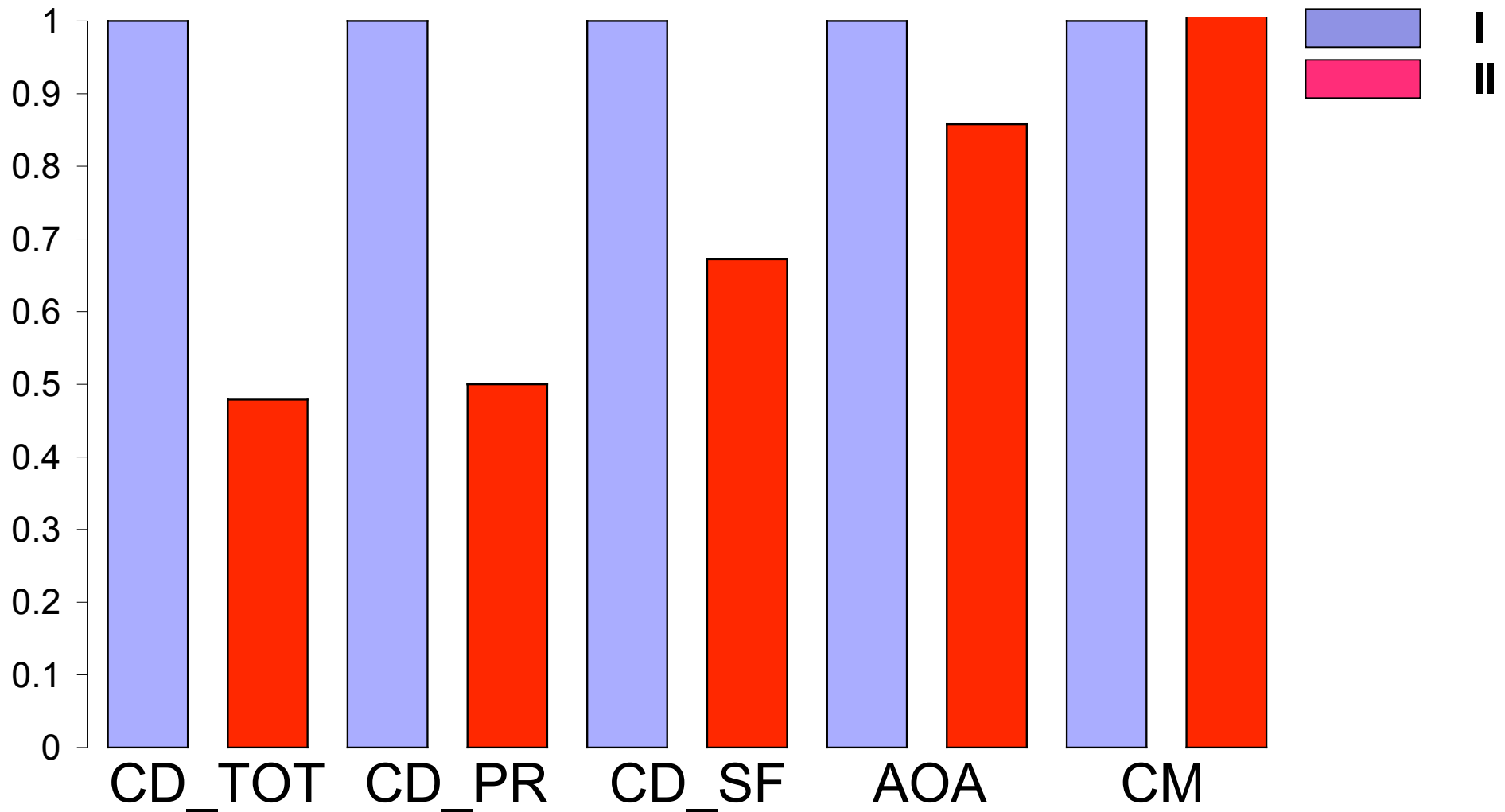
## Case 1



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# STDEV for WB (med. grid)

DPW

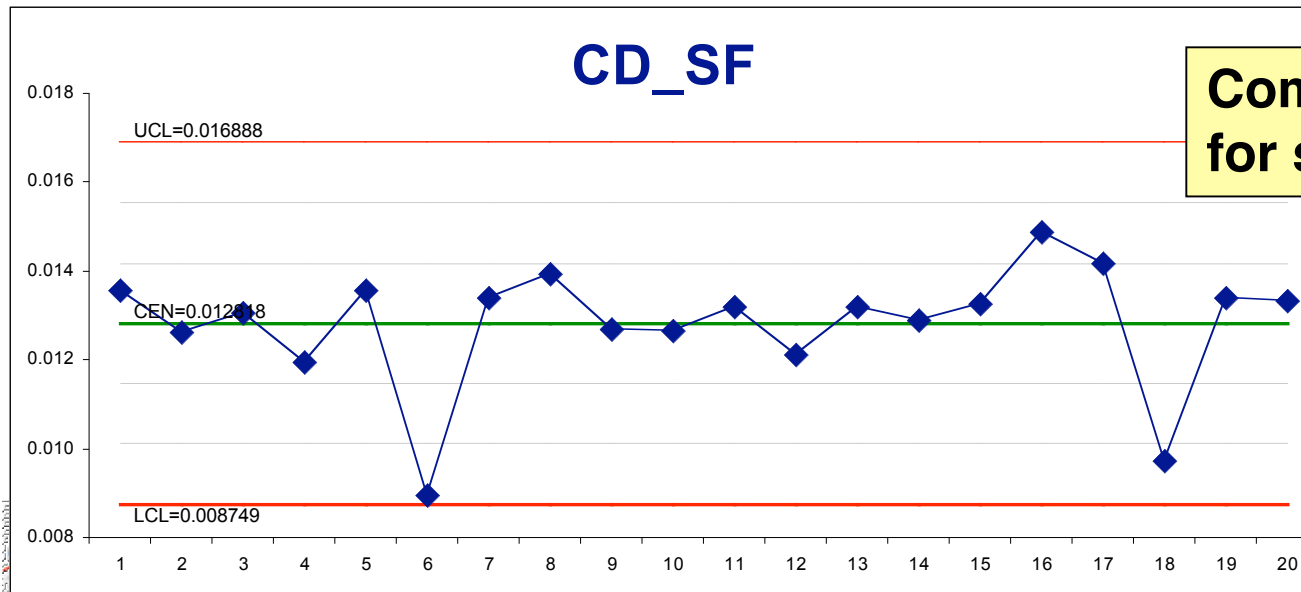
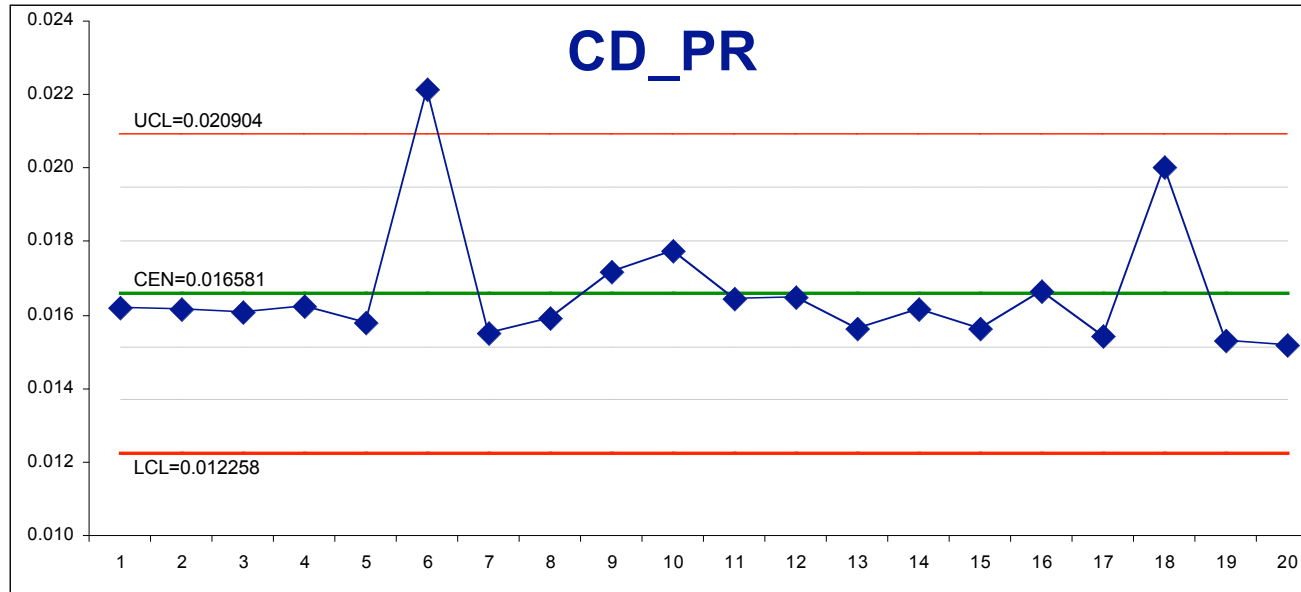


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## Case 1 – Outliers Removed

# DPW II – Case 1 (CD\_PR & CD\_SF)

# DPW



**Compensating “errors”  
for solutions 6 and 18**

- 3:1 reduction in spread of results
- Improvement in dispersion of “core” solutions is more modest
  - Ranged from zero (CM) to 2:1 (CDTOT and CDPR)



# Convergence (???) or “A Tale of Statistics”



# What does convergence look like?

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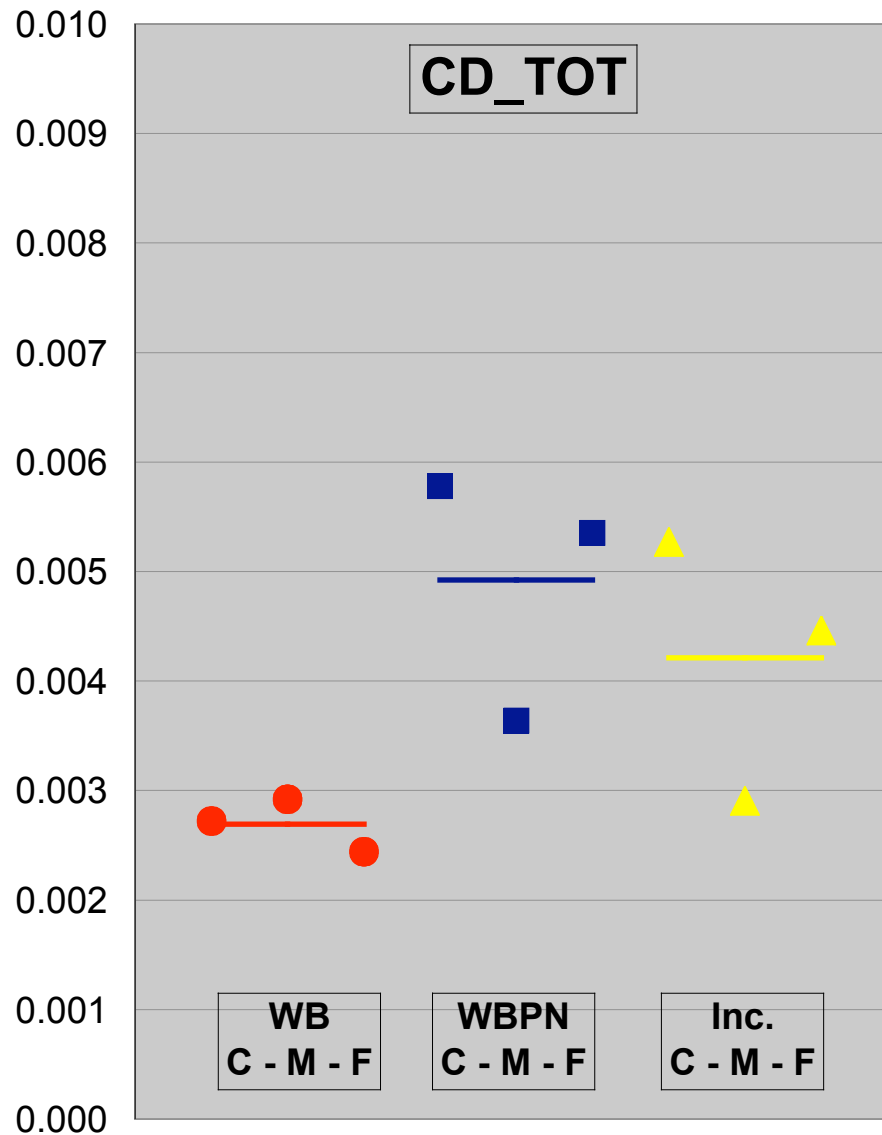
DPW

- **For the collective to show convergence, the following would have to happen:**
  - The ranges for the configurations would have to decrease as the grid “improved”.
  - The scatter (standard deviation) of the “core” solutions would have to decrease as the grid improved.
  - The medians of the core solutions would change asymptotically.



# Convergence of Spread (Ranges)

DPW

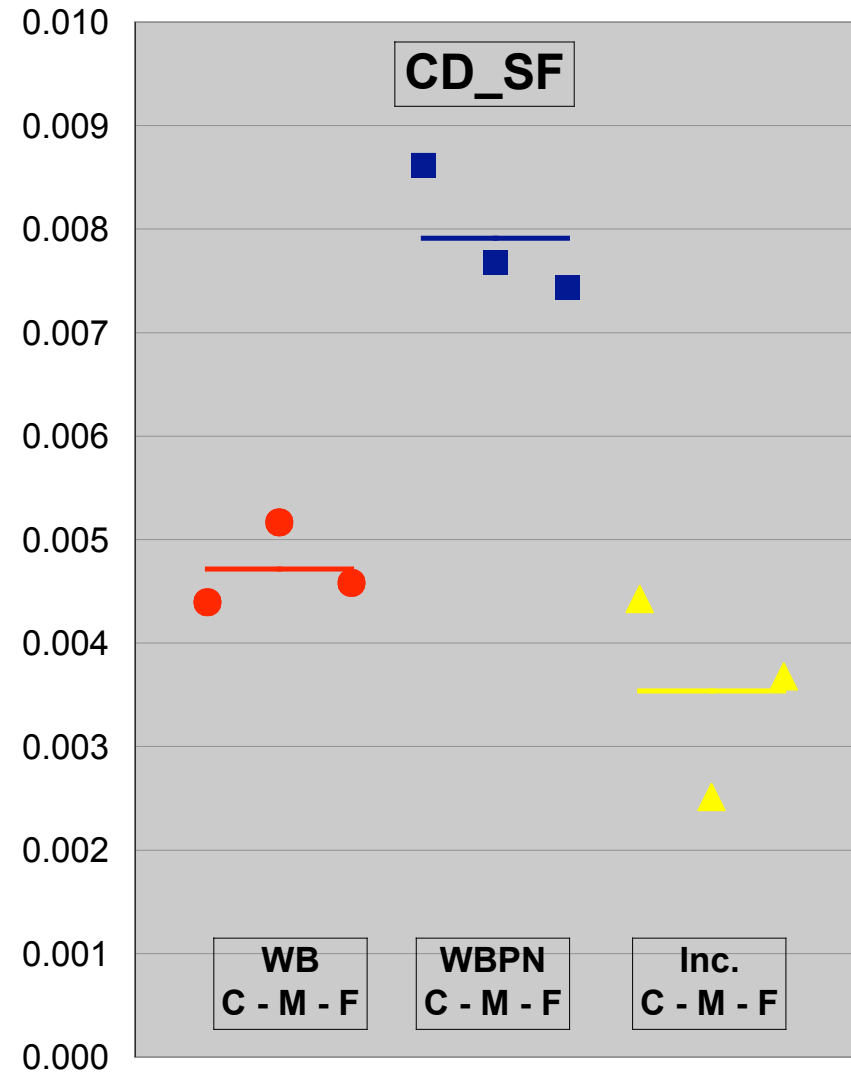
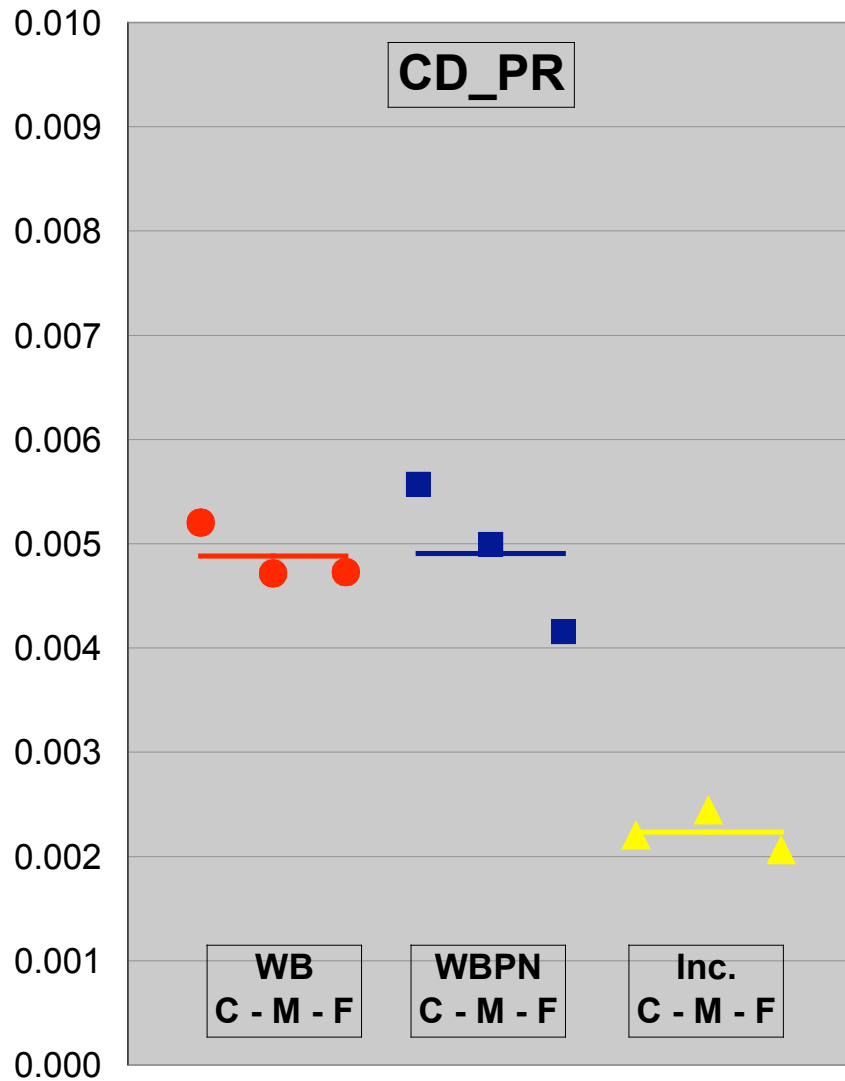


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# Convergence of Spread (Ranges)

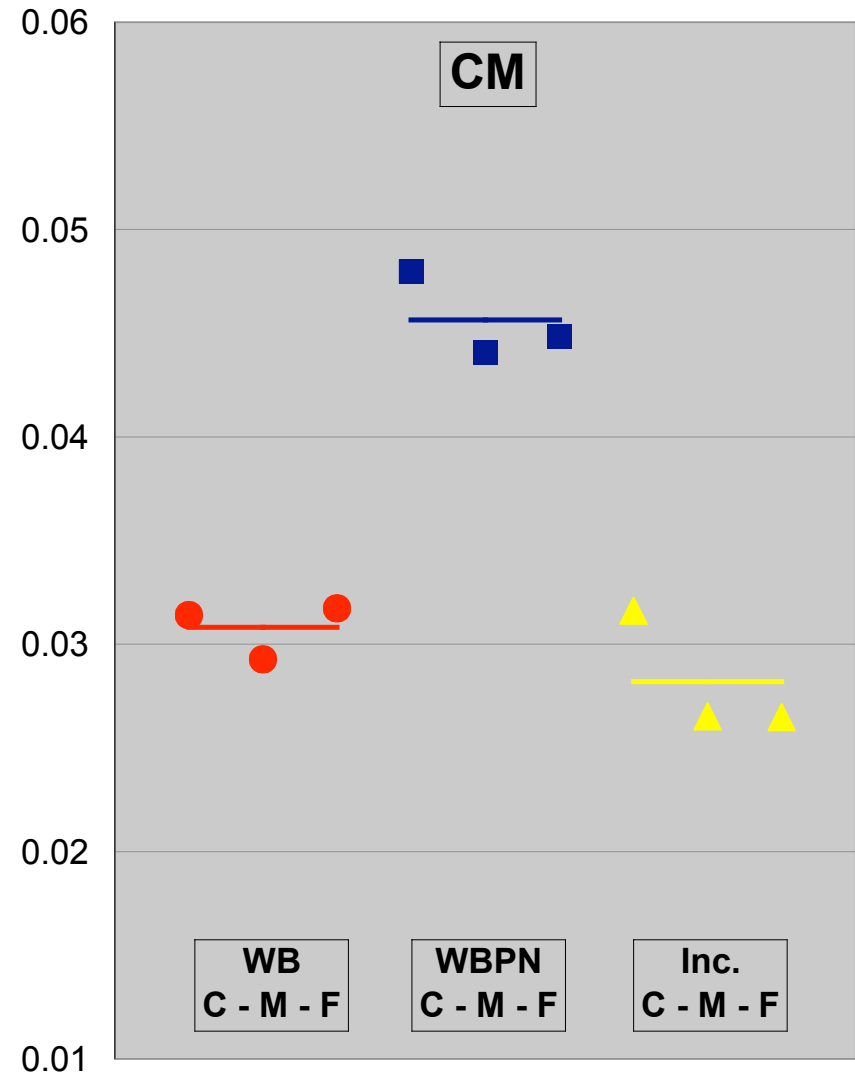
DPW



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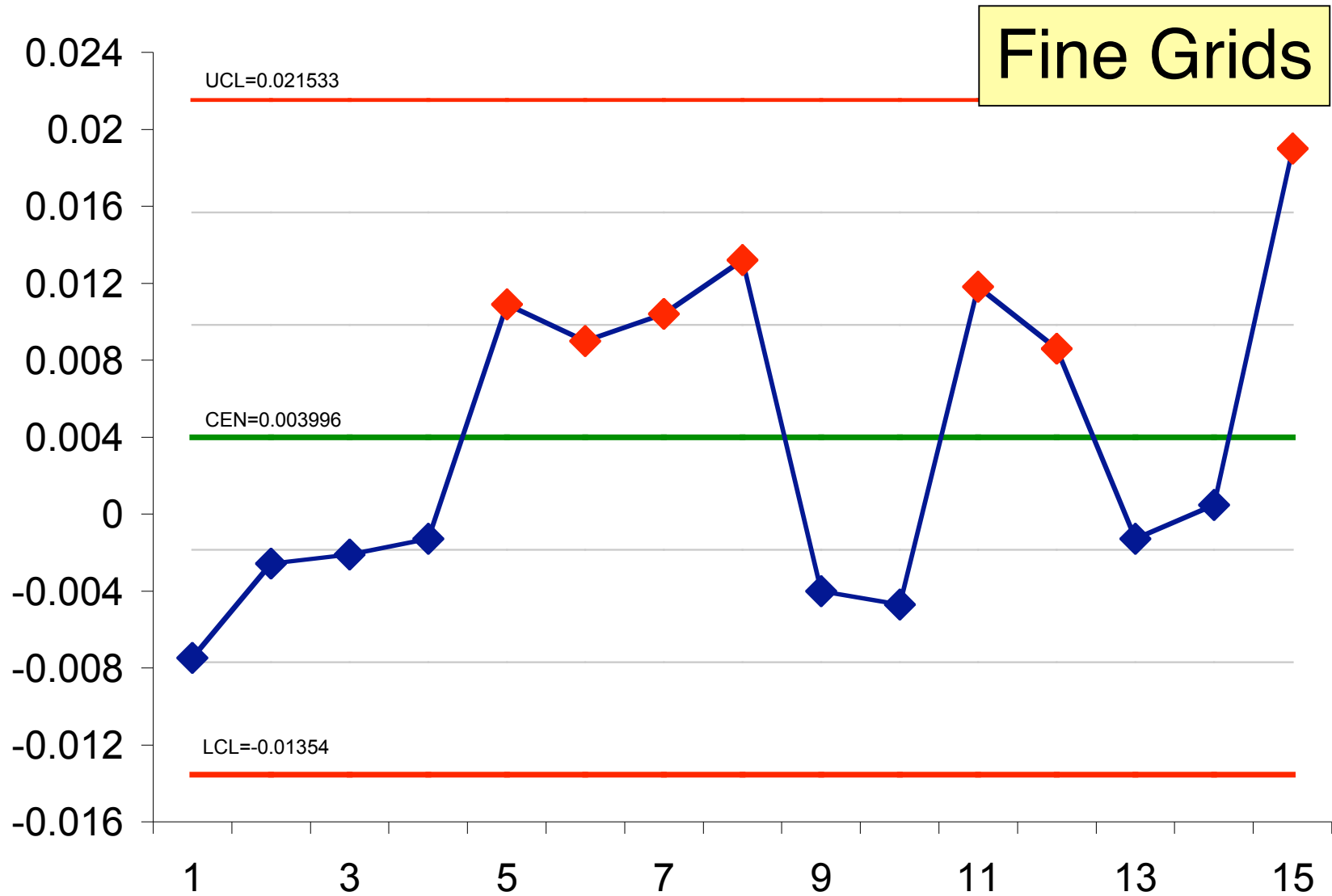
# Convergence of Spread (Ranges)

DPW



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# Two CM-Increment Populations DPW



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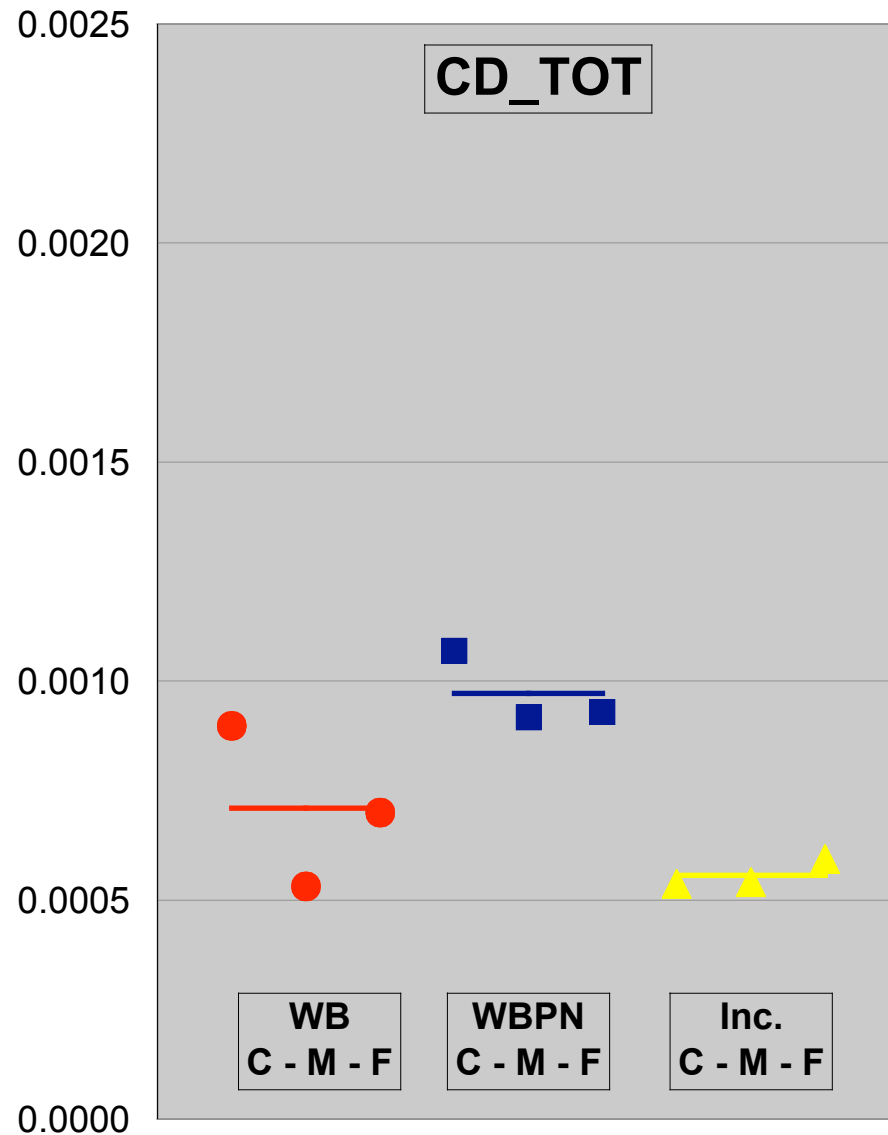
# Removal of outliers for conv. study DPW

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- Control charts revealed that the results from three codes should be removed for analysis of core scatter and medians.
- Statistics for remaining solutions:
  - 12 authors
  - 10 institutions
  - 11 codes

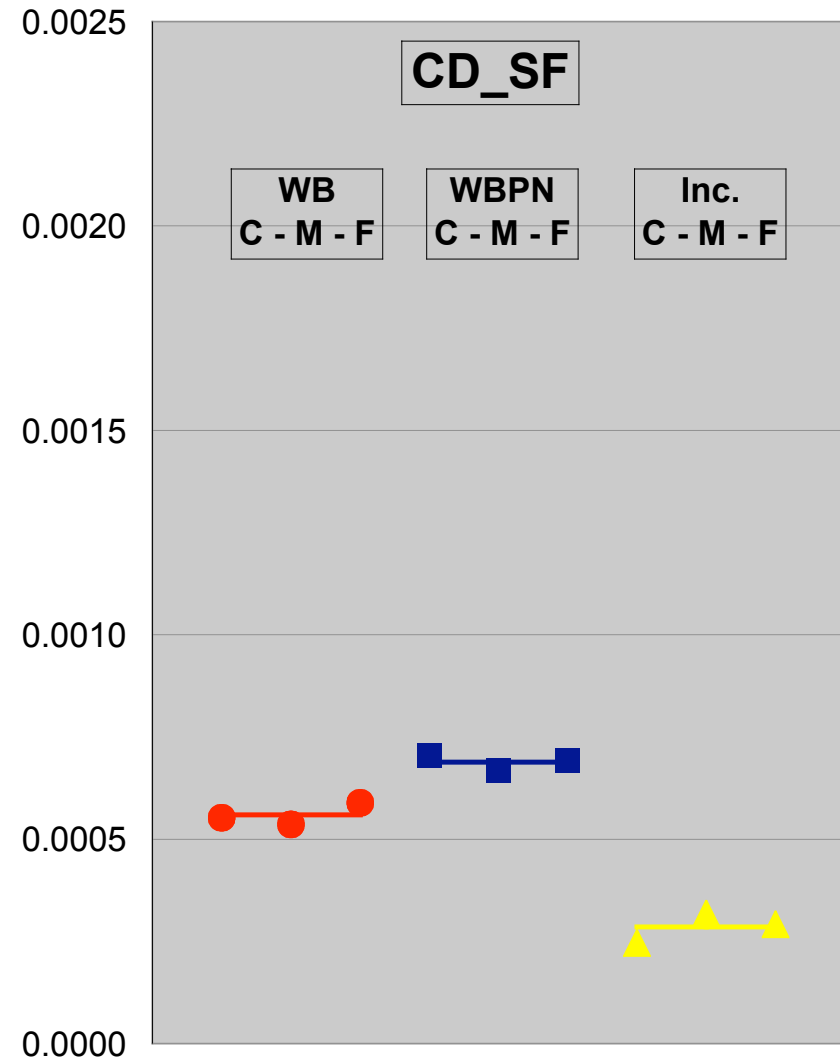
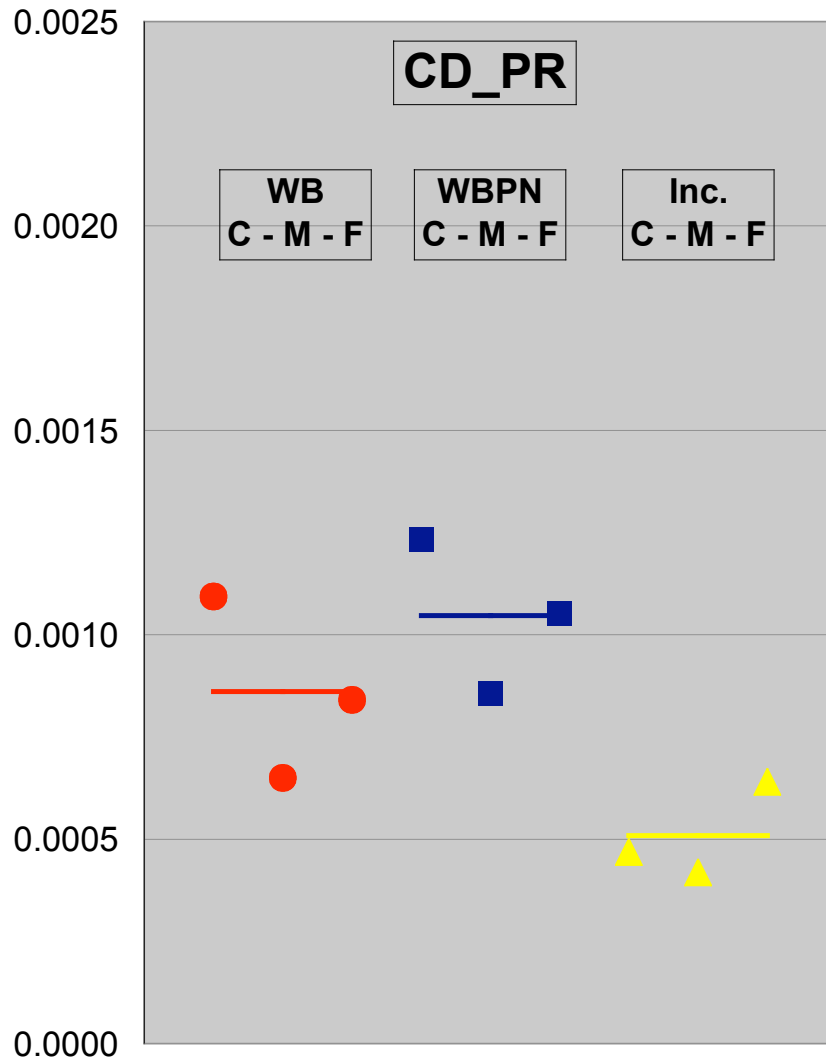


# Convergence of Standard Deviations DPW

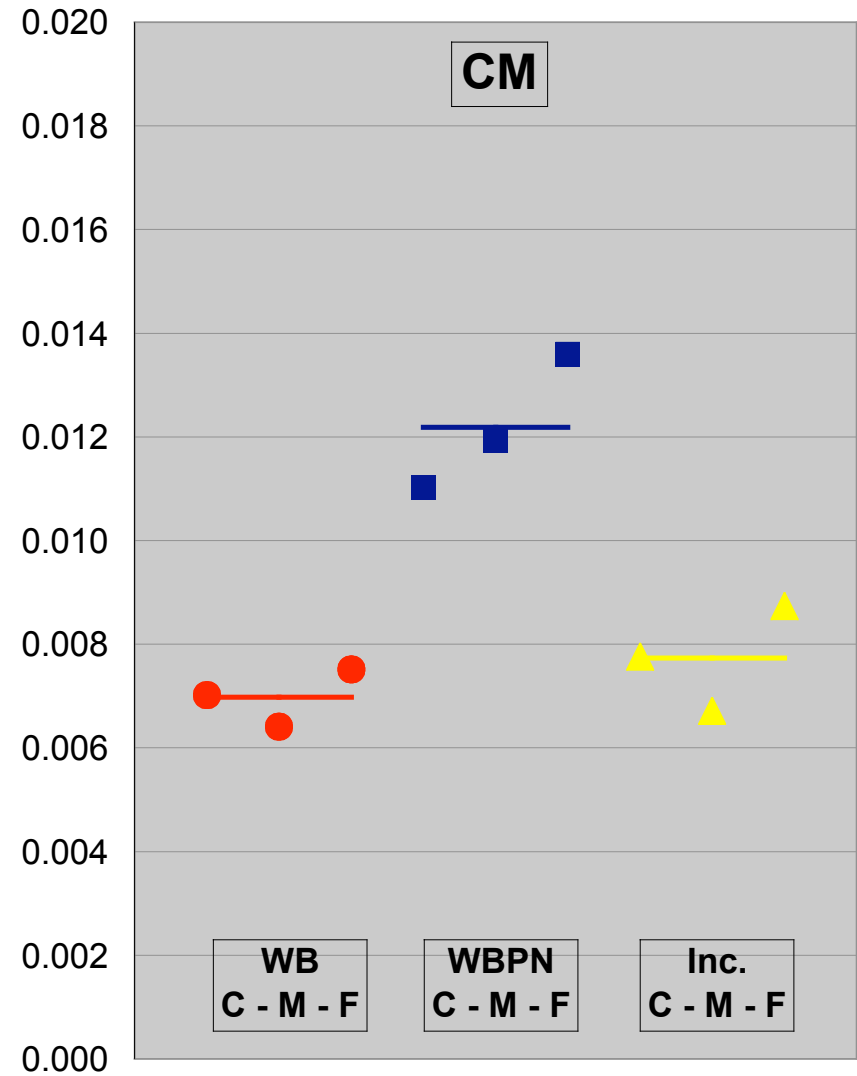
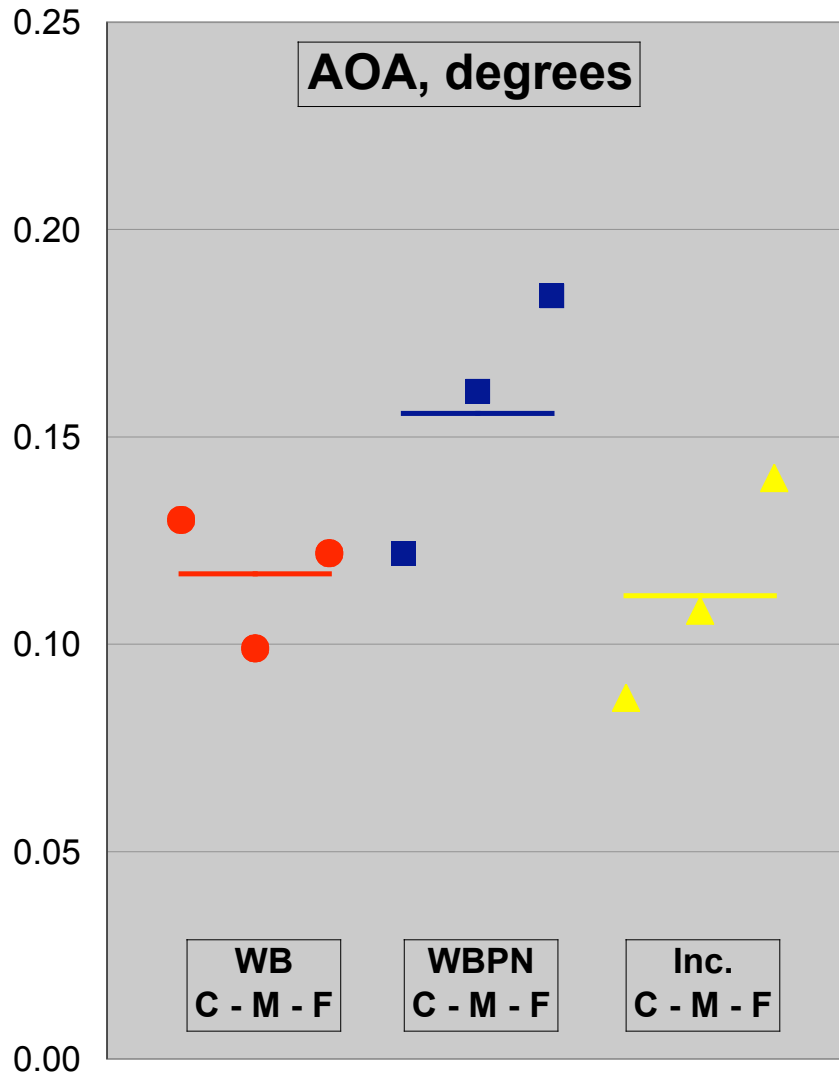


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# Convergence of Standard Deviations **DPW**



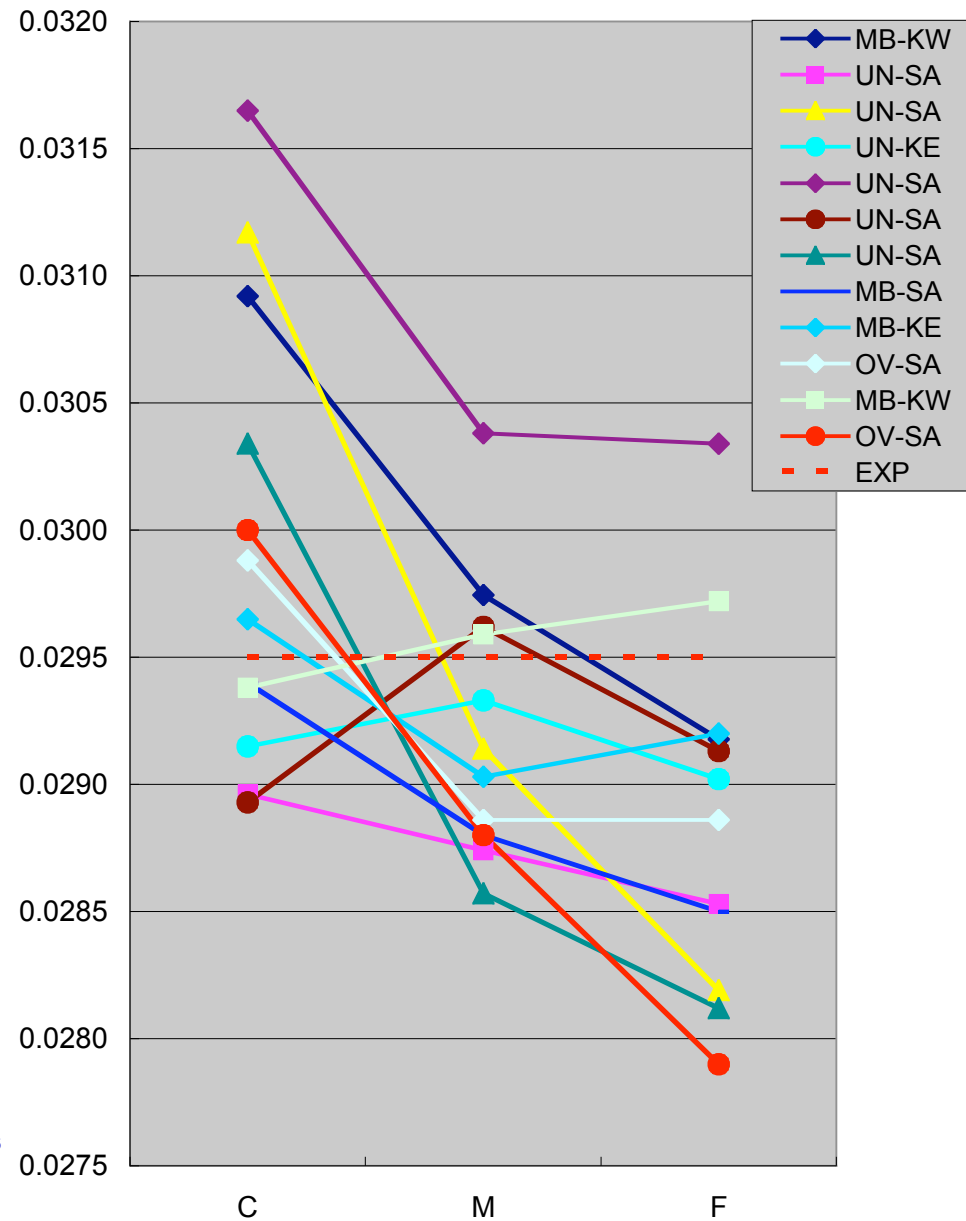
# Convergence of Standard Deviations **DPW**



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# Convergence of CD\_TOT for WB

DPW

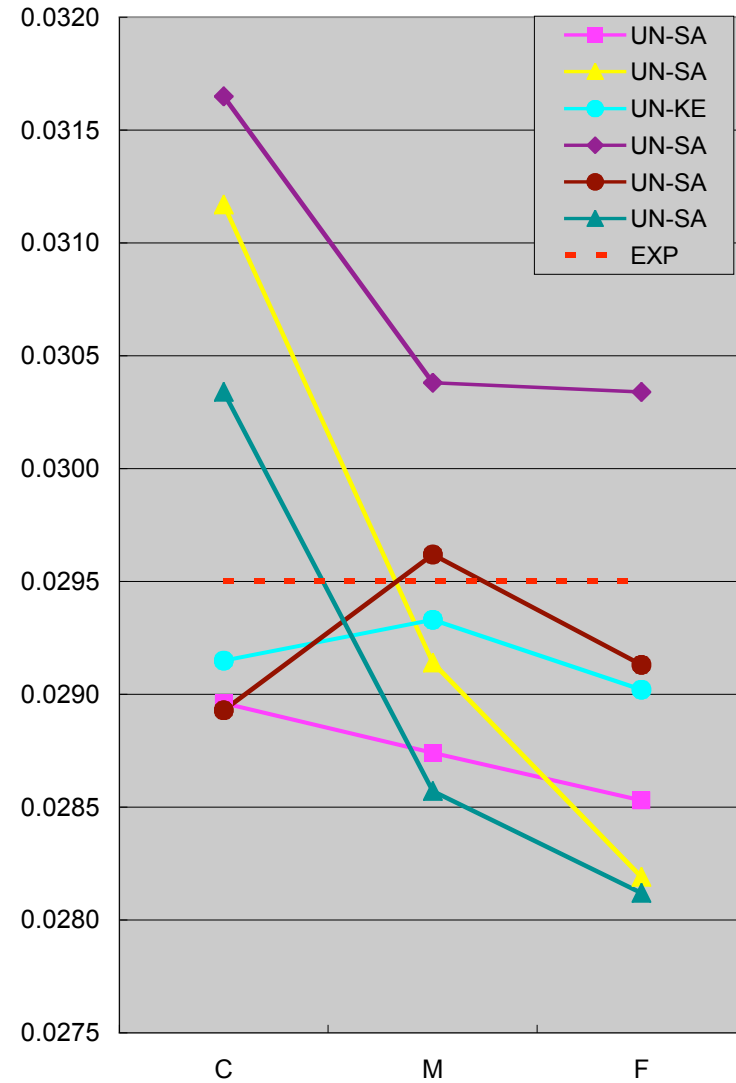
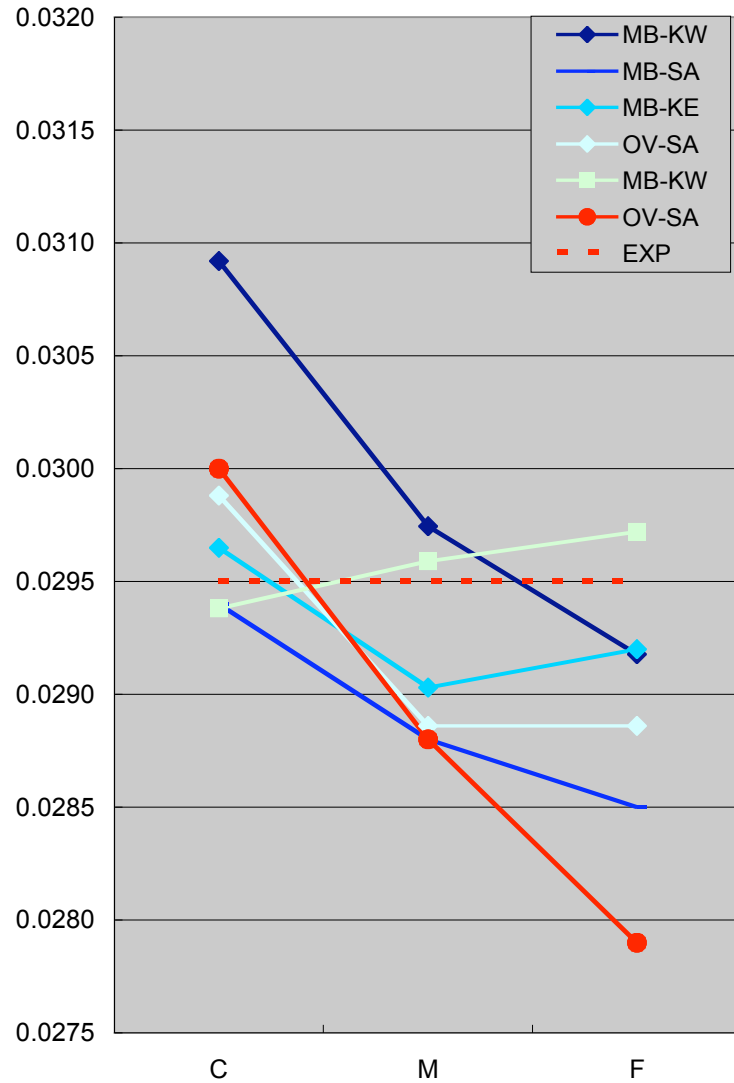


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# Convergence of CD\_TOT for WB

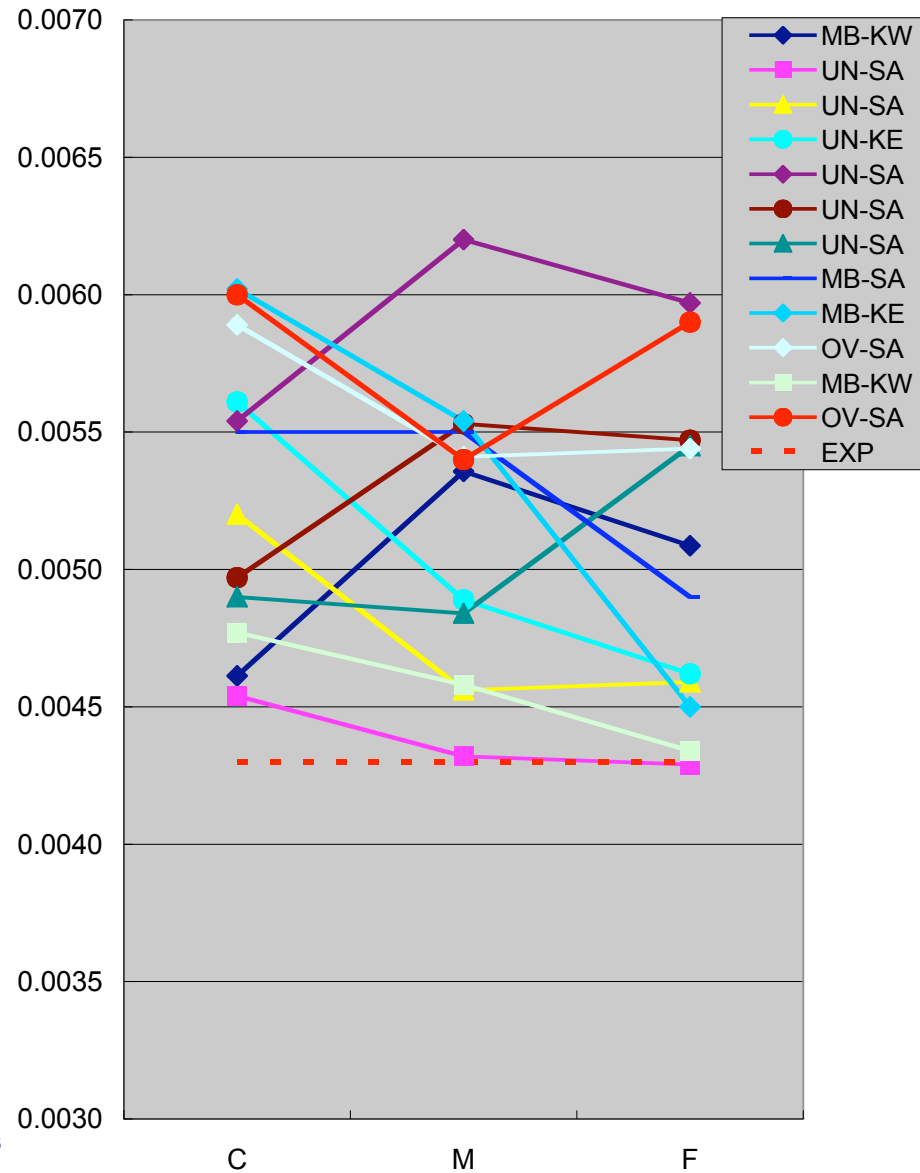
DPW



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# Convergence of CD\_TOT for INCR.

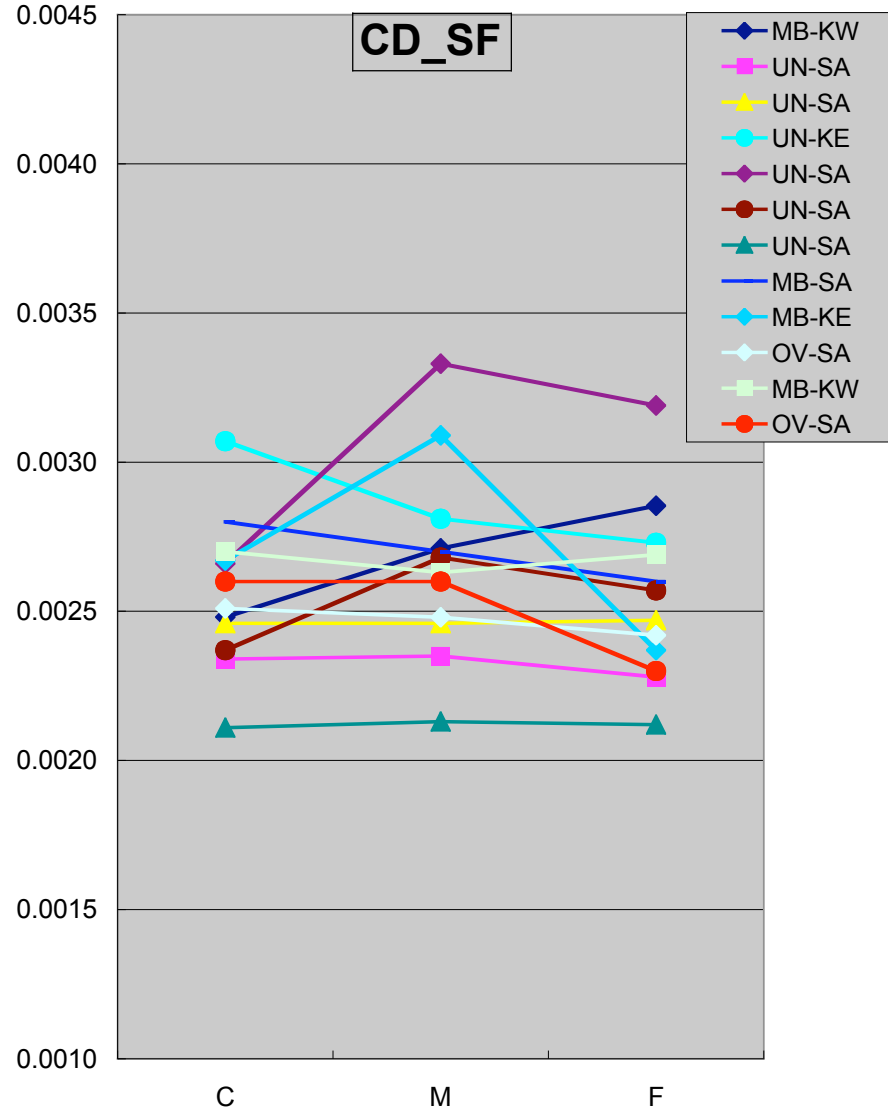
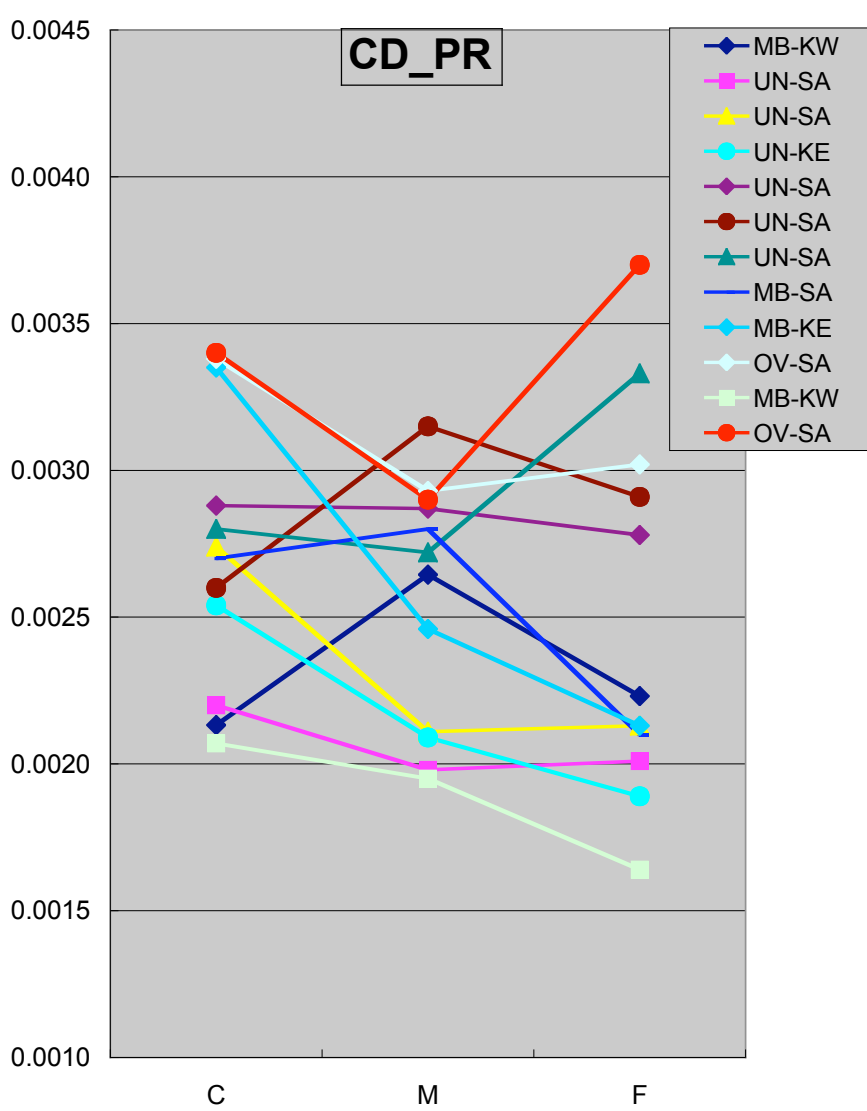
DPW



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# Convergence for INCR.

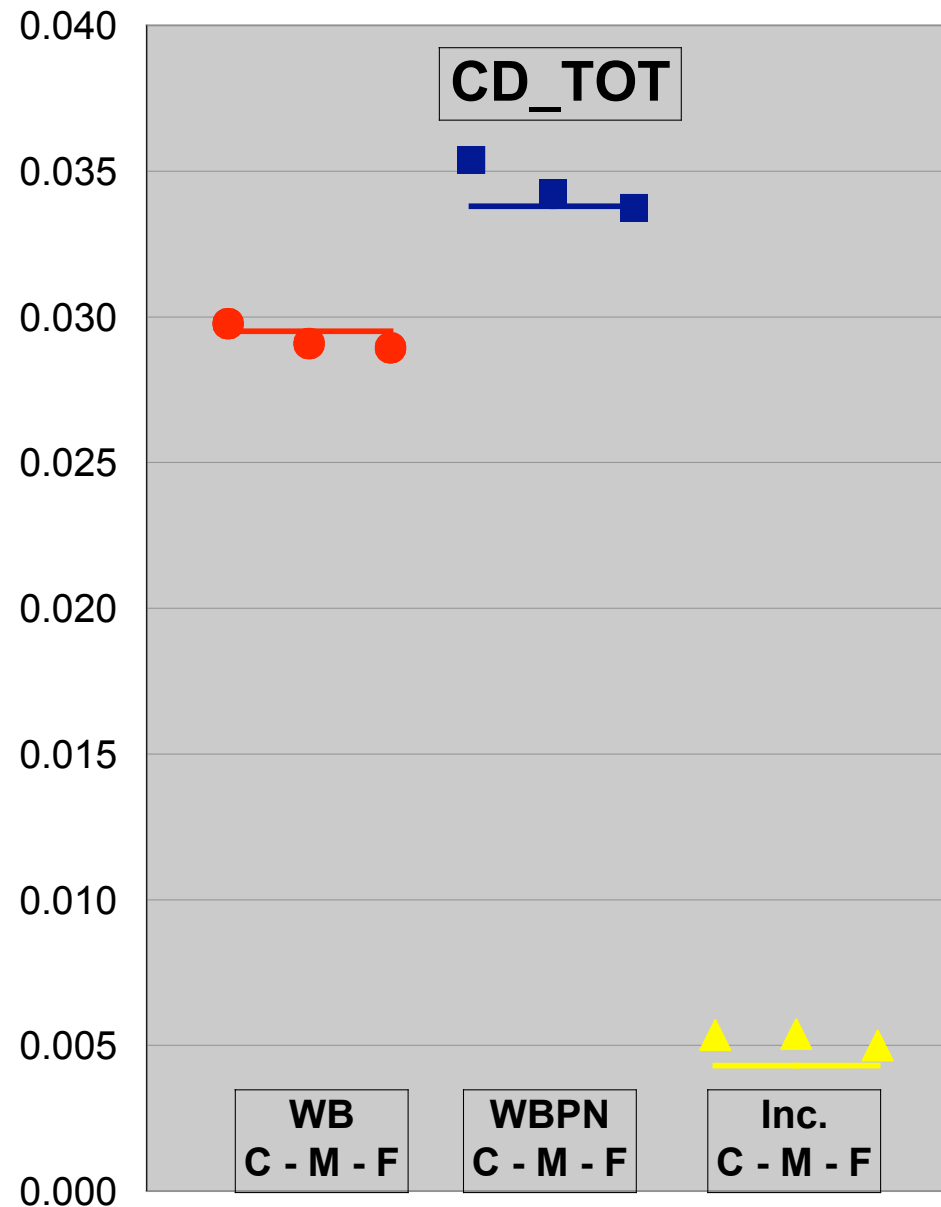
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# Convergence of Medians

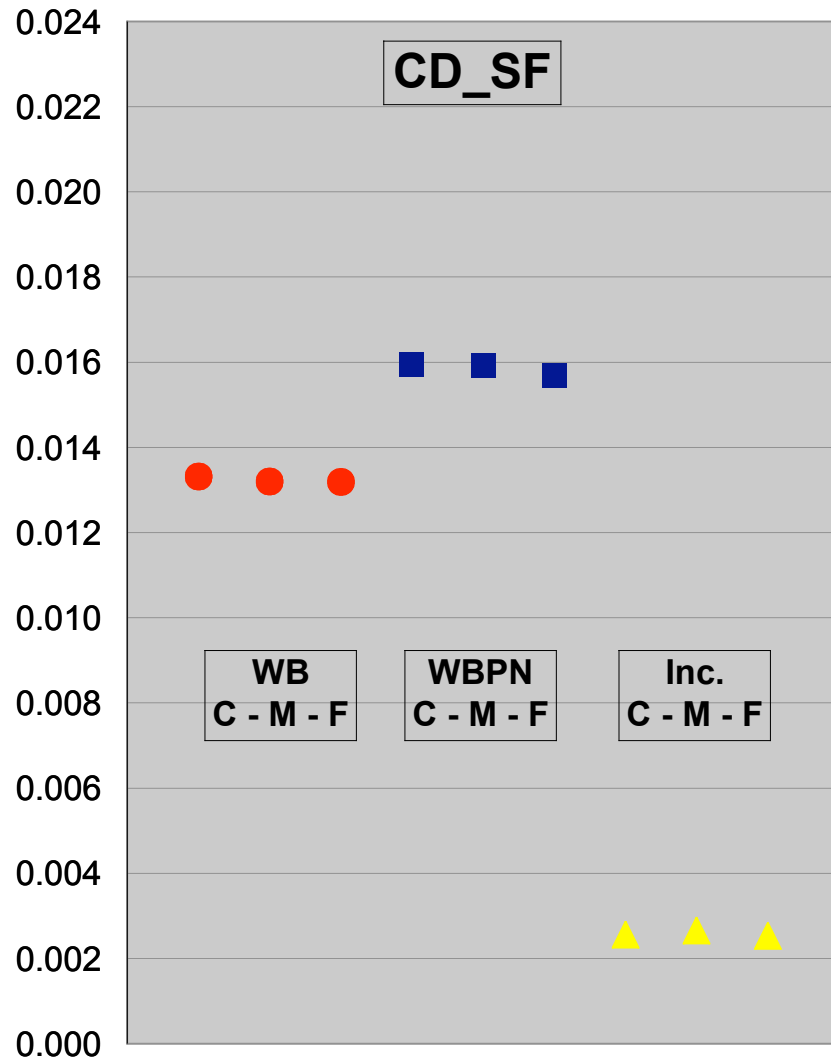
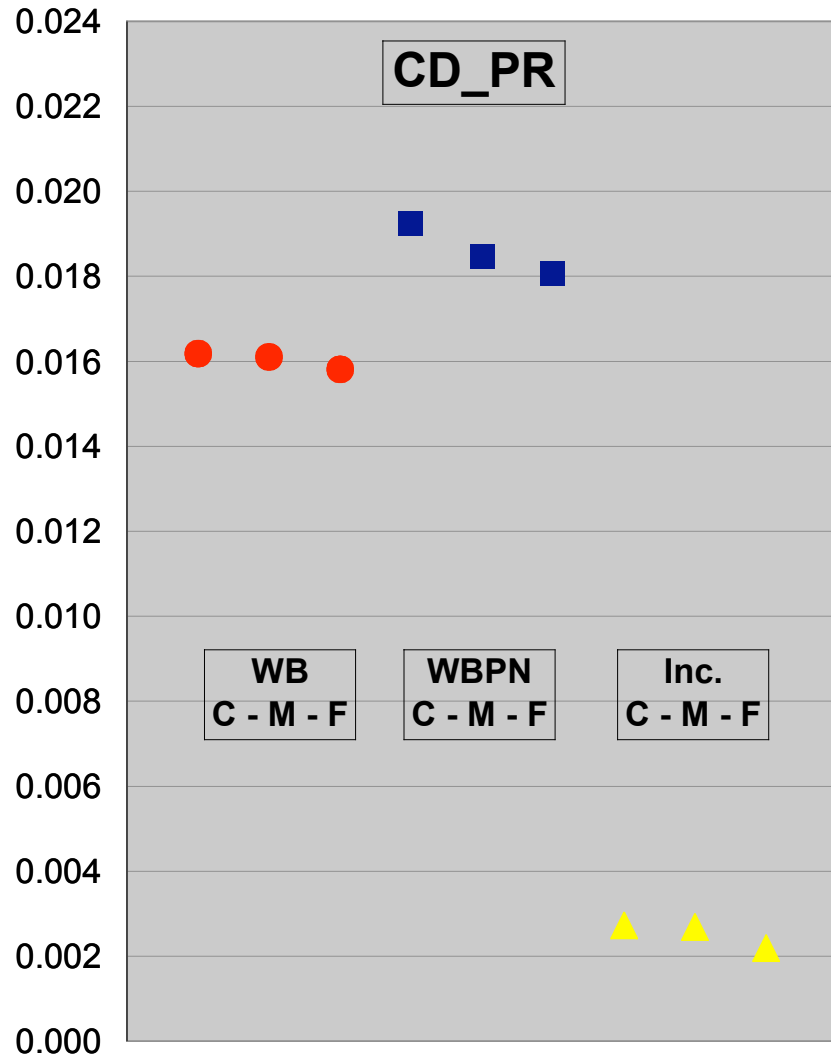
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# Convergence of Medians

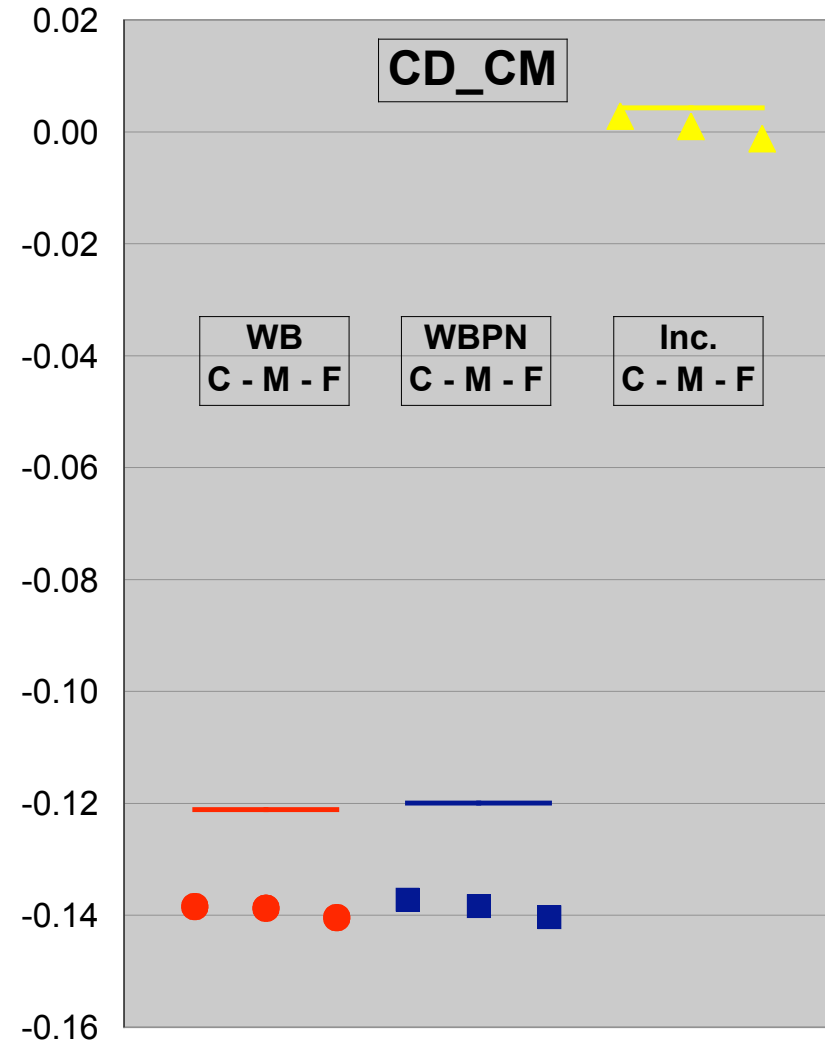
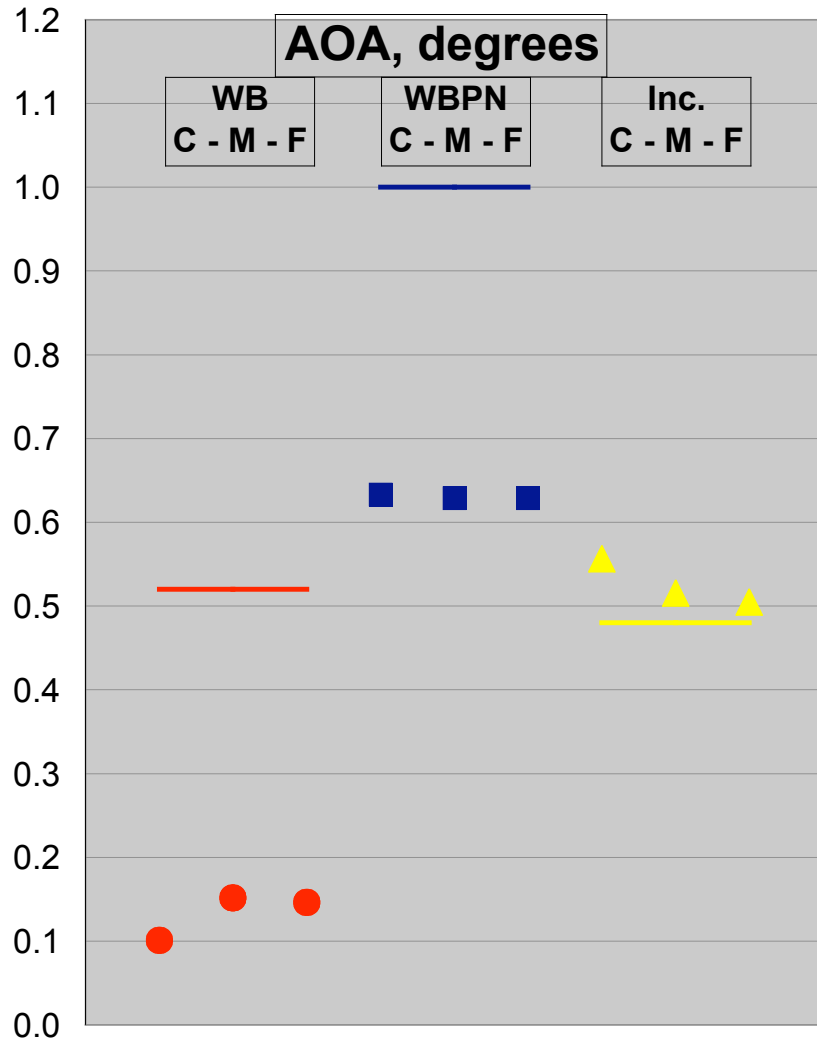
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# Convergence of Medians

DPW



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- For medium grid on wing-body, DPW II results are better than DPW I.
- Regarding grid convergence for the collective:
  - **There is no reduction in spread;**
  - **There is no reduction in core scatter;**
  - **The medians MAY be converging, although it can't be proven with the present results.**
- Increments tend to be considerably better in both scatter and median.
- Much work needs to be done to define what is meant by grid convergence, i.e. how to carry it out.



# Recommendation 1 – Num. Exp. DPW

- For experimental work, it is well known that quality monitoring and improvement requires the following not-so-obvious elements:
  - **A well-defined process (that is actually followed). This generates an operational definition for the output quantities.**
  - **Well-defined sampling from the process output for comparison.**
  - **Statistical monitoring of the samples over time.**
- I believe that grid convergence work should pay strict attention to these three elements. There must be well-defined sampling from a well-defined collective to get the full benefit of the work.





# Recommendation 1 continued DPW

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- In any process in which a fair number of “error” sources are contributing roughly equally but not always with the same values and signs, the behavior of the output will seem random.
- Hence, quality improvement requires isolation of the sources so that their effects can be reduced by process improvement or by correction.
- In order to pick up interactions between sources e.g. turbulence model and grid type/size, the design of the numerical experiment should be done using MDOE.



**Don't use OFAT!**

# Recommendation 2 – Phys. Exp. DPW

- **Give the codes a chance but set them up to fail.**
  - Create a new wind tunnel model which uses a mounting system such that corrections are simple and accurate (e.g. sting and not too big for the walls).
  - Use CFD and body filler to make sure that there are no separation regions for the design point. Note that the definition of “design point” is where the code is supposed to be right.
  - Design the test matrix so that the code can fail in two ways: (1) gracefully and (2) catastrophically. **Statisticians call this “severe testing”.**

