A new wave in fluid dynamics

The issue of uncertainties in experiments and computations

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Introduction

- All experimental set-ups, as well as flight operations, contain sources of uncertainties
- Examples common to EFD and CFD
  - Manufacturing Tolerances
  - Uncertainties on inlet and boundary conditions
- In CFD
  - Uncertainties due to the physical models (Turbulence)

- Experimentalists, designers and CFD simulations should consider the existence of these uncertainties
- New methodologies are required in order to incorporate the presence of uncertainties in the simulation process

- This leads to non-deterministic simulations

Introduction

‘Uncertainties’ (uncontrollable) are to be distinguished from ‘errors’ (controllable)

- Experiments
  - Errors due to the limited accuracy of the instrumentation
- CFD
  - Numerical errors due to truncation errors of schemes
  - Grid dependence and sensitivities

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What is required

1. Identification of the most relevant uncertainties

2. Quantification of the uncertainties
   - Provide a probability density distribution (pdf) for each uncertainty

3. For CFD only:
   - estimate the propagation of the uncertainties by mathematical and algorithmic methods for the treatment of differential equations containing stochastic input data and model parameters (new area dealing with Stochastic PDE’s)

4. Evaluation of the uncertainty impact and **domain of confidence** of the measured or predicted quantities

Identification of uncertainties

This has to rely on the experience and expert knowledge of designers and experimentalists

Identified uncertainties

- Operational conditions: Inlet or exit flow conditions
- Geometrical uncertainties
  - Tip clearance
  - Fabrication tolerances on geometry; Leading edge, TE; blade shapes
  - Blade inlet or outlet angles
  - Roughness
- Modeling uncertainties, such as turbulence models

Propagation of the uncertainties

- Innovative mathematical and algorithmic methods have to be developed for the treatment of differential equations containing stochastic input parameters and model parameters

- One of the most interesting methods is the *Polynomial Chaos Method (PCM)*

- In this approach, the randomness of the flow solution is represented by pdf's of the different variables at every point and instant of time.

Polynomial Chaos method

- The basis of the chaos expansion is to approximate the random process by a complete and orthogonal polynomial basis in terms of random variables (Wiener 1938).

- All random variables are represented as

$$u = u\left(\vec{x}, t, \xi_1, \ldots, \xi_n\right)$$

$$u\left(x, t, \vec{\xi}\right) = \sum_{k=0}^{P} u_k\left(x, t\right) \Psi_k(\vec{\xi})$$

where $\Psi_k(\xi)$ are orthogonal polynomials

- The orthogonal polynomials are associated to types of random variables

Two options are available

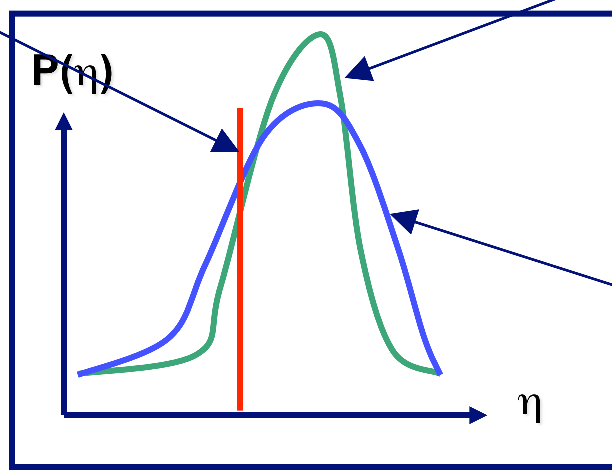
- Non-Intrusive PCM—NI-PCM
  - The non-intrusive method uses an existing deterministic flow solver as a black box. The deterministic solver is run for several sample points in a stochastic space and the outcome of the runs are post-processed to extract the relevant statistical descriptors.
  - Probabilistic Collocation Method, Probabilistic Radial Basis Function; Chaos Collocation

- Intrusive PCM—I-PCM
  - The PC expansion is inserted in the flow equations and the outcome is a set of coupled governing equations for the PCM-coefficients.
  - This coupling is intrusive as the implementation requires modifying the deterministic flow solver.

Representative output

- Comparison of results of different approaches
- Elaboration of consequences and strategies for design process (CPU- and user time, accuracy)

Result of deterministic CFD simulations for mean value of uncertainty parameter as input

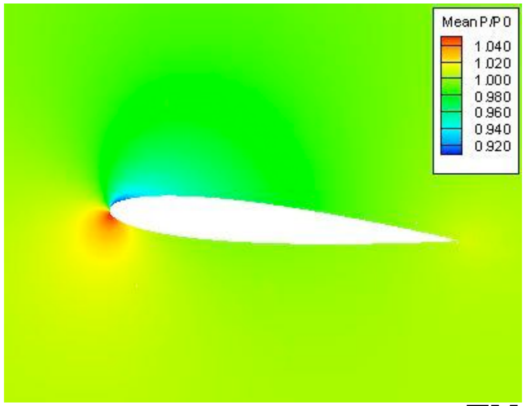


pdf of non-deterministic CFD simulations for given pdf of uncertainty parameter as input (IPCM)

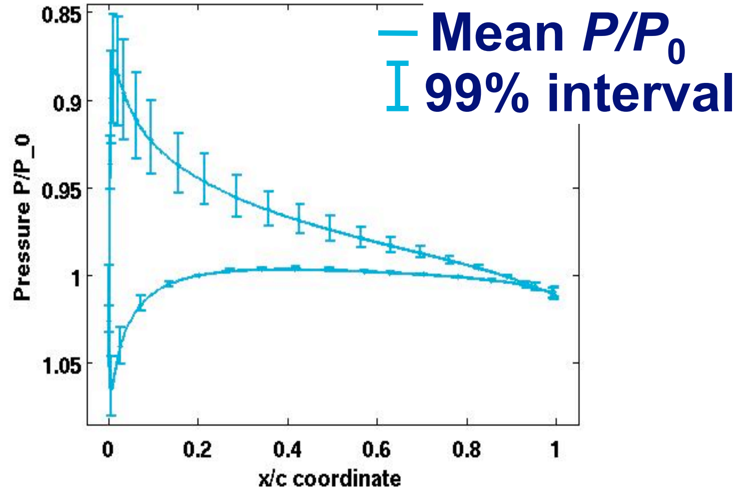
pdf of deterministic CFD simulations for randomly sampled uncertainty parameter as input (NIPCM)

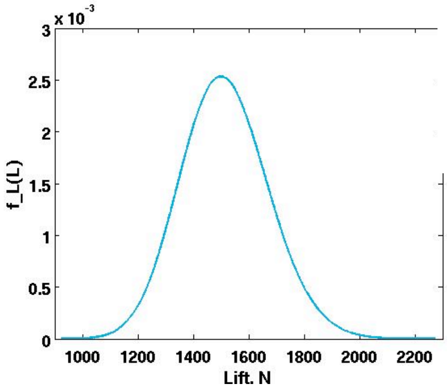
Operational uncertainty

NACA0012 airfoil, Re=3*10^6, incidence= 3 deg.
Mean M=0.3, coefficient of variation 5%

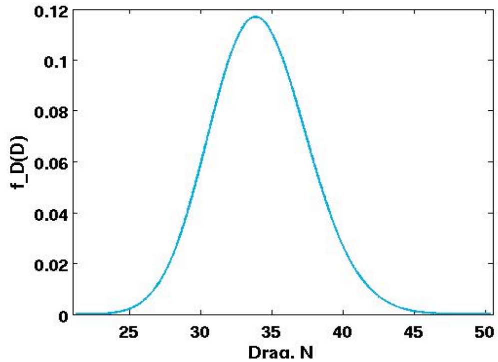


Flow solver: FINE/Hexa™





μ_L = 1515 N
σ_L = 158 N
CV_L = 0.10

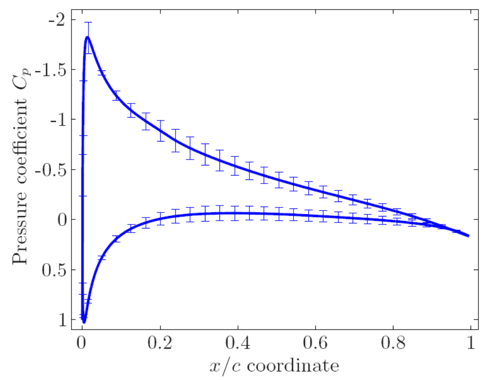


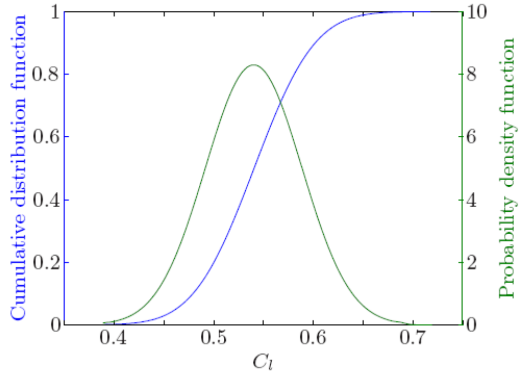
μ_D = 34.1 N
σ_D = 3.42 N
CV_D = 0.10

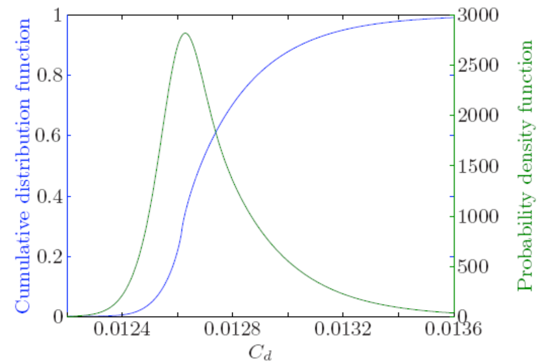
NACA0012 airfoil, Re=3*10[6], incidence: 3 deg.

- Uncertain relative thickness, with standard deviation of 0.425% and truncated normal distribution and
- Uncertain camber *c*, mean camber 0%, with standard deviation of 0.4472%, and truncated normal

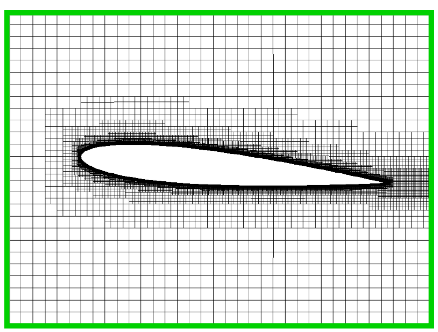
FINE/Hexa computation

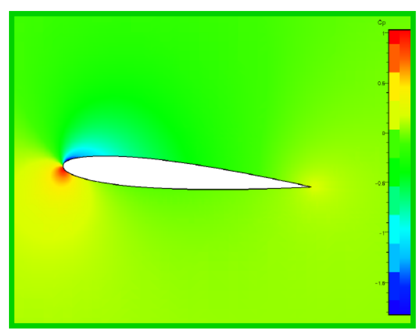






The error bars indicate the standard deviation of the airfoil surface pressure coefficient. As part of the output, pdf's of lift and drag are produced



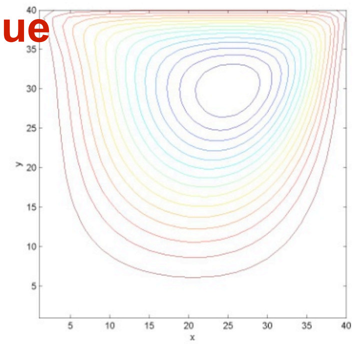


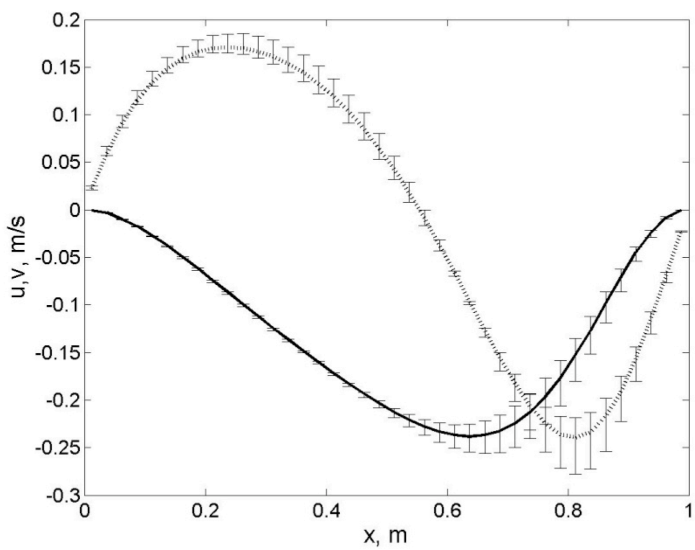
Model uncertainties

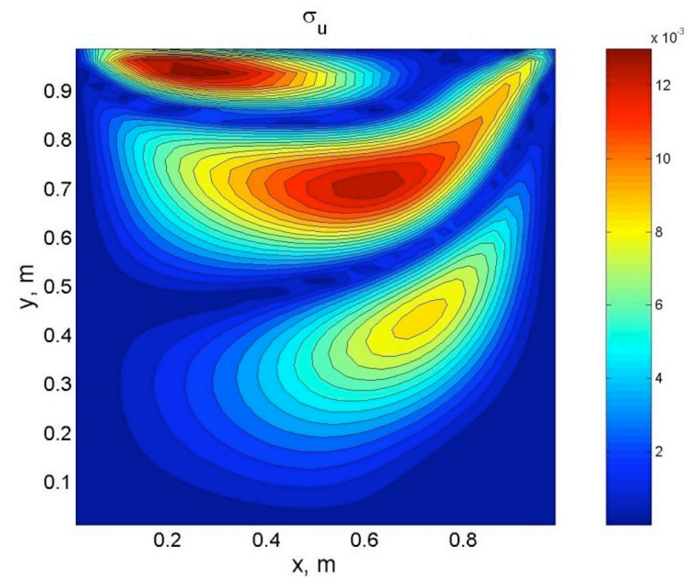
- <u>Intrusive PCM</u>:
  - lid-driven cavity problem: **viscosity has been considered uncertain with a standard deviation of 10% with respect to its mean value**
  - flow solver used: FINE/Turbo
  - left figure: the distributions of the mean velocities

  along the horizontal centerline in a lid driven cavity problem

  (u – solid line, v – dashed line)
  - right figure: the standard deviation field of the horizontal velocity







Rotor 37 Operational uncertainty

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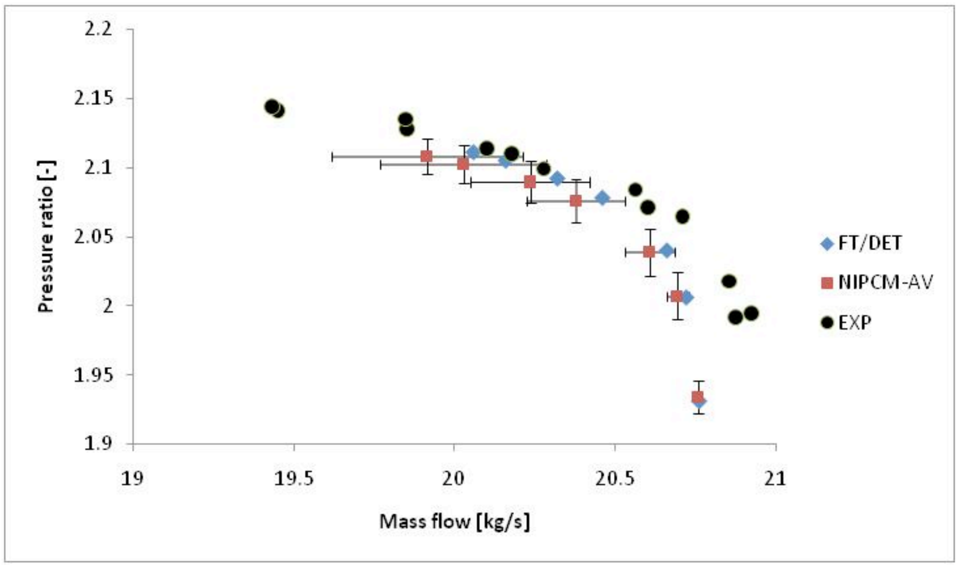
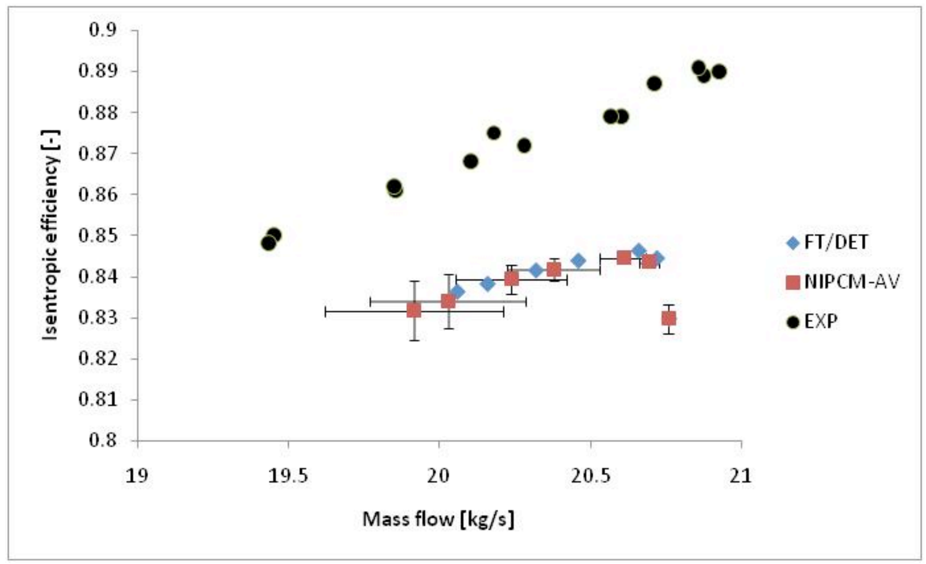
- Non-deterministic simulation—2nd order Polynomial Chaos method
- 4% uncertainty, Gaussian, on downstream pressure
- Notice that the average probabilistic values differ from the deterministic values





Pressure ratio and Efficiency versus mass flow compressor map: error bars represent variation of +/- σ/2 (σ=standard deviation)

Recommendation

For the experimentalists

- Provide as input
  - Uncertainties (pdf's) on the operational conditions of the WT (pdf's of incident Mach and flow angle)
  - Deviations between WT models and CAD geometries
  - Any other quantity susceptible to add uncertainties (e.g. WT wall suction, if any)
- As output
  - Experimental data with estimated 99% domain of confidence

For CFD

- Produce calculations taking into account the experimental identified uncertainties

More information

EU project NODESIM-CFD http://www.nodesim.eu

Workshop on uncertainty quantification in CFD

Two main test cases: RAE airfoil and Rotor 37

with prescribed operational and geometrical uncertainties

Brussels, 29-30 October 2009